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Pseudo Symmetric Ideals In Ternary Semigroups

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ABSTRACT : *In this paper the terms pseudo symmetric ideals, pseudo symmetric ternary semigroups, semipseudo symmetric ideals and semipseudo symmetric ternary semigroups. It is proved that for any pseudo symmetric ideal A in a ternary semigroup T, for any natural number n,* $a_1a_2 \ldots a_{n-1}a_n \in A$ *if and only if* $< a_1 > < a_2 > \ldots\ldots\ldots < a_n > \subseteq A$. *It is proved that every completely semiprime ideal of a ternary semigroup is a pseudo symmetric ideal. Further it is proved that an ideal A of a ternary semigroup is (1) completely prime iff A is prime and pseudo symmetric, (2) completely semiprime iff A is semiprime and pseudo symmetric. It is also proved that every prime ideal P minimal relative to containing a pseudo symmetric ideal A in a ternary semigroup T is completely prime and hence every prime ideal P minimal relative to containing a completely semiprime ideal A in a ternary semigroup T is completely prime. It is proved that every pseudo commutative ternary semigroup, ternary semigroup in which every element is a mid unit, are pseudo symmetric ternary semigroups. It is proved that every pseudo symmetric ideal of a ternary semigroup is a semipseudo symmetric ideal. It is also proved that every semiprime ideal P minimal relative to containing a semipseudo symmetric ideal A of a ternary semigroup is completely semiprime. If A is a semipseudo symmetric ideal of a ternary semigroup T, then (1)* A_1 *= the intersection of all completely prime ideals of T containing A, (2)* A_1' *= the intersection of all minimal completely prime ideals of T containing A, (3)* A_1'' *= the minimal completely semiprime ideal of T relative to containing A, (4)* $A_2 = \{x \in T : x^n \in A$ *for some odd natural number n},(5)* A_3 *= the intersection of all prime ideals of T containing A, (6)* A_3' *= the intersection of all minimal prime ideals of T containing A, (7)* A_3'' *= The minimal semiprime ideal of relative to containing A (8)* $A_4 = \{x \in T : <x>^n \subseteq A$ *for some odd natural number n}, are equivalent. If A is an ideal in a ternary semigroup then it is proved that (1) A is completely semiprime, A is semiprime and pseudo symmetric, A is semiprime and semipseudo symmetric, are equivalent, and (2) A is completely prime, A is prime and pseudo symmetric, A is prime and semipseudo symmetric are also equivalent. If M is a maximal ideal of a ternary semigroup T with* $M_4 \neq T$ *then it is proved that M is completely prime, M is completely semiprime, M is pseudo symmetric and M is semipseudo symmetric, are equivalent. It is proved that a ternary semigroup is a semipseudo symmetric ternary semigroup iff every principal ideal is a semipseudo symmetric ideal. If T is a semipseudo symmetric ternary semigroup then it is proved that the conditions; T is strongly archimedean, T is archimedean, T has no proper completely prime ideals, T has no proper completely semiprime ideals, T has no proper prime ideals and T has no proper semiprime ideals are equivalent. Further it is proved that an element is semisimple iff it is intra regular in a pseudo symmetric ternary semigroup. It is proved that every globally idempotent ternary semigroup having maximal ideals contains semisimple elements. It is proved that every nontrivial maximal ideal of a semipseudo symmetric ternary semigroup is prime. If T is semipseudo symmetric ternary semigroup and contains a non-trivial maximal ideal then it is shown that T contains semisimple elements. If T is a semipseudo symmetric ternary semigroup, then it is proved that (1)* $T = \{ a \in T : \sqrt{< a >} \neq T\}$ *is either empty or a prime ideal, (2) T\S is either empty or an archimedean subsemigroup of T.*

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I. INTRODUCTION

RAMAKOTAIAH and ANJANEYULU [1] introduced the notions of pseudo symmetric ideals in semigroups, pseudo symmetric semigroups and exhibit some examples and some classes of pseudo symmetric semigroups. KRISHNA MURTHY and ARUL DOSS[11] introduced the notions of pseudo symmetric Γ-ideals in Γ-semirings. MADHUSUDHANA RAO, ANJANEYULU and GANGADHARA RAO[16] introduce and made a study on pseudo symmetric ideals in semigroups.

ANJANEYULU [1], [2] introduced a class of semigroups called as semipseudo symmetric semigroups which contains commutative semigroups, pseudo commutative semigroups, quasi commutative semigroups, normal semigroups and idempotent semigroups (bands) and obtained an analogue of KRULL's theorem in commutative rings [24] for semipseudo symmetric semigroups. ANJANEYULU obtained necessary conditions for existence of semisimple elements in a semipseudo symmetric semigroup and characterize maximal ideals in an archimedian semipseudo symmetric semigroup. MADHUSUDHANA RAO, ANJANEYULU and GANGADHARA RAO[17] introduce semipseudo symmetric Γ-semigroups and obtain the results regarding semipseudo symmetric semigroups to semipseudo symmetric Γ-semigroups.

II. PRELIMINARIES

DEFINITION 2.1 : Let T be a non-empty set. Then T is said to be a ***ternary semigroup*** if there exist a mapping from T×T×T to T which maps $(x_1, x_2, x_3) \rightarrow [x_1x_2x_3]$ satisfying the condition

: $[[x_1x_2x_3]x_4x_5] = [x_1[x_2x_3x_4]x_5] = [x_1x_2[x_3x_4x_5]]$ $\forall$ $x_i \in$ T, $1 \le i \le 5$.

DEFINITION 2.2 : A ternary semigroup T is said to be ***commutative*** provided for all a,b,c$\in$ T, we have abc = bca = cab = bac = cba = acb.

DEFINITION 2.3 : A ternary semigroup T is said to be ***quasi commutative*** provided for each *a, b, c* ∈ T, there exists a natural number *n* such that abc = b^n ac = bca = c^n ba = cab = a^n cb.

DEFINITION 2.4 : A ternary semigroup T is said to be ***normal*** provided abT = Tab $\forall$ *a,b*$\in$ T.

DEFINITION 2.5 : A ternary semigroup T is said to be ***left pseudo commutative*** provided *abcde = bcade = cabde = bacde = cbade = acbde* $\forall$ *a,b,c,d,e* $\in$ T.

DEFINITION 2.6: A ternary semigroup T is said to be a ***lateral pseudo commutative*** ternary semigroup provide *abcde = acdbe = adbce = acbde = adcbe = abdce* for all *a,b,c,d,e* $\in$ T.

DEFINITION 2.7 : A ternary semigroup T is said to be ***right pseudo commutative*** provided *abcde = abdec = abecd = abdce = abedc = abced* $\forall$ *a,b,c,d,e* $\in$ T.

DEFINITION 2.8: A ternary semigroup T is said to be ***pseudo commutative***, provided T is a left pseudo commutative, right pseudo commutative and lateral pseudo commutative ternary semigroup.

DEFINITION 2.9 : An element *a* of ternary semigroup T is said to be ***left identity*** of T provided *aat* = *t* for all *t*$\in$ T.

NOTE 2.10 : Left identity element *a* of a ternary semigroup T is also called as ***left unital element.***

DEFINITION 2.11 : An element *a* of a ternary semigroup T is said to be a ***lateral identity*** of T provided *ata = t* for all *t* $\in$ T.

NOTE 2.12 : Lateral identity element *a* of a ternary semigroup T is also called as ***lateral unital element.***

DEFINITION 2.13 : An element *a* of a ternary semigroup T is said to be a ***right identity*** of T provided *taa = t* $\forall$ *t*$\in$ T.

NOTE 2.14 : Right identity element *a* of a ternary semigroup T is also called as ***right unital element.***

DEFINITION 2.15 : An element *a* of a ternary semigroup T is said to be a ***two sided identity*** of T provided *aat = taa = t* $\forall$ *t*$\in$ T.

NOTE 2.16 : Two-sided identity element of a ternary semigroup T is also called as ***bi-unital element***.

DEFINITION 2.17 : An element *a* of a ternary semigroup T is said to be an ***identity*** provided *aat = taa = ata = t* $\forall$ *t*$\in$ T.

NOTE 2.18 : An identity element of a ternary semigroup T is also called as ***unital element.***

NOTE 2.19 : An element *a* of a ternary semigroup T is said to be an ***identity*** of T then *a* is left identity , lateral identity and right identity of T.

NOTATION 2.20 : Let T be a ternary semigroup. If T has an identity, let T^1 = T and if T does not have an identity , let T^1 be the ternary semigroup T with an identity adjoined usually denoted by the symbol 1.

DEFINITION 2.21 : An element a of a ternary semigroup T is said to be ***regular*** if there exist $x, y \in$ T such that $axaya = a$.
DEFINITION 2.22 : A ternary semigroup T is said to be ***regular ternary semigroup*** provided every element is regular.
DEFINITION 2.23 : An element a of a ternary semigroup T is said to be ***left regular*** if there exist $x, y \in$ T such that $a = a^3xy$.
DEFINITION 2.24 : An element a of a ternary semigroup T is said to be ***lateral regular*** if there exist $x, y \in$ T such that $a = xa^3y$.
DEFINITION 2.25 : An element a of a ternary semigroup T is said to be ***right regular*** if there exist $x, y \in$ T such that $a = xya^3$.
DEFINITION 2.26 : An element a of a ternary semigroup T is said to be ***intra regular*** if there exist $x, y \in$ T such that $a = xa^5y$.
DEFINITION 2.27 : An element a of a ternary semigroup T is said to be ***completely regular*** if there exist $x, y \in$ T such that $axaya = a$ and $axa = xaa = aax = aya = yaa = aay = axy = yxa = xay = yax$.
DEFINITION 2.28 : An element a of a ternary semigroup T is said to be a ***completely regular ternary semigroup*** provided every element in T is completely regular.
DEFINITION 2.29: An element a of a ternary semigroup T is said to be a ***mid unit*** provided $xayaz = xyz$ for any $x, y, z \in$ T.
DEFINITION 2.30 : An element a of a ternary semigroup T is said to be ***semisimple*** provided $a \in \langle a\rangle^3$, that is $\langle a\rangle^3 = \langle a\rangle$
DEFINITION 2.31 : A ternary semigroup T is said to be a ***semisimple ternary semigroup*** provided every element in T is semisimple.
DEFINITION 2.32 : A ternary semigroup T is said to be an ***archimedian ternary semigroup*** provided for any $a, b \in$ T there exists a odd natural number n such that $a^n \in \mathrm{T}b\mathrm{T}$.
DEFINITION 2.33 : A ternary semigroup S is said to be a ***strongly archimedean ternary semigroup*** provided for any $a, b \in$ T, there is a odd natural number n such that $\langle a\rangle^n \subseteq \langle b\rangle$.
THEOREM 2.34 : Every strongly archimedean ternary semigroup is an archimedean ternary semigroup.
DEFINITION 2.35 : An element a of a ternary semigroup T is said to be an ***idempotent*** element provided $a^3 = a$.
DEFINITION 2.36 : An element a of a ternary semigroup T is said to be a ***proper idempotent*** element provided a is an idempotent which is not the identity of T if identity exists.
DEFINITION 2.37 : A ternary semigroup T is said to be an ***idempotent ternary semigroup*** of ***ternary band*** provided every element of T is an idempotent.
DEFINITION 2.38 : An ideal A of a ternary semigroup T is called a ***globally idempotent ideal*** if $\mathrm{A}^n = \mathrm{A}$ for all odd natural number n.
DEFINITION 2.39 : A ternary semigroup T is said to be a ***globally idempotent ternary semigroup*** if $\mathrm{T}^n = \mathrm{T}$ for all odd natural number n.
DEFINITION 2.40 : A nonempty subset A of a ternary semigroup T is said to be ***left ideal*** of T if $b, c \in$ T, $a \in$ A implies $bca \in$ A.
NOTE 2.41 : A nonempty subset A of a ternary semigroup T is said to be a left ideal of T if and only if TTA $\subseteq$ A.
DEFINITION 2.42 : A nonempty subset of a ternary semigroup T is said to be a ***lateral ideal*** of T if $b, c \in$ T , $a \in$ A implies $bac \in$ A.
NOTE 2.43 : A nonempty subset of A of a ternary semigroup T is a lateral ideal of T if and only if TAT $\subseteq$ A.
DEFINITION 2.44 : A nonempty subset A of a ternary semigroup T is a ***right ideal*** of T if $b, c \in$ T , $a \in$ A implies $abc \in$ A
NOTE 2.45 : A nonempty subset A of a ternary semigroup T is a right ideal of T if and only if ATT $\subseteq$ A.
DEFINITION 2.46 : A non-empty subset A of a ternary semigroup T is said to be ***ternary ideal*** or simply an ***ideal*** of T if $b, c \in$ T , $a \in$ A implies bca $\in$ A, $bac \in$ A, $abc \in$ A.
NOTE 2.47 : A nonempty subset A of a ternary semigroup T is an ideal of T if and only if it is left ideal, lateral ideal and right ideal of T.
DEFINITION 2.48 : An ideal A of a ternary semigroup T is said to be a ***proper ideal*** of T if A is different from T.
DEFINITION 2.49 : An ideal A of a ternary semigroup T is said to be a ***maximal ideal*** provided A is a proper ideal of T and is not properly contained in any proper ideal of T.
DEFINITION 2.50 : An ideal A of a ternary semigroup T is said to be a ***completely prime ideal*** of T provided $x, y, z \in$ T and $xyz \in$ A implies either $x \in$ A or $y \in$ A or $z \in$ A.

DEFINITION 2.51 : An ideal A of a ternary semigroup T is said to be a ***prime ideal*** of T provided X,Y,Z are ideals of T and XYZ $\subseteq$ A $\Rightarrow$ X $\subseteq$ A or Y $\subseteq$ A or Z $\subseteq$ A.
DEFINITION 2.52 : An ideal A of a ternary semigroup T is said to be a ***completely semiprime ideal*** provided $x \in$ T, $x^n \in$ A for some odd natural number n >1 implies $x \in$ A.
DEFINITION 2.53 : An ideal A of a ternary semigroup T is said to be ***semiprime ideal*** provided X is an ideal of T and $X^n \subseteq$ A implies X ⊆ A.
THEOREM 2.54: If T is a globally idempotent ternary semigroup then every maximal ideal of T is a prime ideal of T.
NOTATION 2.55 : If A is an ideal of a ternary semigroup T, then we associate the following four types of sets.

A_1 = The intersection of all completely prime ideals of T containing A.

A_2 = $\{x \in T: x^n \in A$ for some odd natural numbers $n\}$

A_3 = The intersection of all prime ideals of T containing A.

A_4 = $\{x \in T: < x >^n \subseteq$ A for some odd natural number $n\}$

THEOREM 2.56 : If A is an ideal of a ternary semigroup T, then $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$.
COROLLARY 2.57 : If an ideal A of a ternary semigroup T is completely semiprime then $x, y, z \in$ T, $xyz \in$ A $\Longrightarrow$ $< x > < y > < z > \subseteq$ A.
THEOREM 2.58 : Let A be a prime ideal of a ternary semigroup T. If A is completely semiprime ideal of T then A is completely prime.
THEOREM 2.59 : The nonempty intersection of any family of completely prime ideals of a ternary semigroup is completely semiprime.
THEOREM 2.60 : An ideal A of a ternary semigroup T is semiprime iff X is an ideal of T, $X^3 \subseteq$ A implies X ⊆ A.
THEOREM 2.61 : Every prime ideal of a ternary semigroup is semiprime.
THEOREM 2.62 : Every completely semiprime ideal of a ternary semigroup is semiprime.
COROLLARY 2.63 : An ideal Q of a semigroup T is a semiprime ideal iff Q is the intersection of all prime ideals of T contains Q.
THEOREM 2.64 : If A is a completely semiprime ideal of a ternary semigroup T, then $x, y, z \in$ T, $xyz \in$ A implies that xyTT$z \subseteq$ A and xTT$yz \subseteq$ A.

III. PSEUDO SYMMETRIC IDEALS

DEFINITION 3.1 : An ideal A of a ternary semigroup T is said to be ***pseudo symmetric*** provided $x, y, z \in$ T, $xyz \in$ A implies $xsytz \in$ A for all $s, t \in$ T.
THEOREM 3.2 : Let A be a pseudo symmetric ideal in a ternary semigroup T and $a, b, c \in$ T. Then $abc \in$ A if and only if $<a> <b><c> \subseteq$ A.
***Proof* :** Suppose $< a > < b >< c > \subseteq$ A. $abc \in < a > < b > < c > \subseteq$ A $\Rightarrow abc \in$ A.
Conversely suppose that $abc \in$ A. Let $t \in < a > < b > < c >$.
Then $t = s_1as_2bs_3cs_4$ where $s_1, s_2, s_3, s_4 \in T^1$.
$abc \in$ A, $s_2, s_3 \in T^1$, A is pseudo symmetric ideal $\Rightarrow as_2bs_3c \in$ A $\Rightarrow s_1as_2bs_3cs_4 \in$ A $\Rightarrow t \in$ A.
Therefore $< a > < b > < c > \subseteq$ A.
COROLLARY 3.3 : Let A be any pseudo symmetric ideal in a ternary semigroup T and $a_1, a_2 ...., a_n \in$ T where n is an odd natural number. Then $a_1a_2....a_n \in$ A if and only if $<a_1> < a_2>....<a_n> \subseteq$ A.
***Proof* :** Clearly if $<a_1> < a_2>....<a_n> \subseteq$ A, then $a_1a_2....a_n \in$ A where n is an odd natural number.
Conversely suppose that $a_1a_2....a_n \in$ A where n is an odd natural number.
Let $t \in <a_1> < a_2>....<a_n>$. Then $t = s_1a_1 s_2a_2....a_ns_{n+1}$, where $s_i \in T^1$, $i = 1,2,...n+1$.
$a_1a_2....a_n \in$ A, A is pseudo symmetric ideal $\Rightarrow s_1a_1 s_2a_2....a_ns_{n+1} \in$ A and hence $t \in$ A.
Therefore $<a_1> < a_2>....<a_n> \subseteq$ A.
COROLLARY 3.4: Let A be a pseudo symmetric ideal in a ternary semigroup T, then for any odd natural number n, $a^n \in$ A if and only if $<a>^n \subseteq$ A.
***Proof* :** The proof follows from corollary 3.3, by taking $a_1 = a_2 = ... a_n = a$.
COROLLARY 3.5 : Let A be a pseudo symmetric ideal in a ternary semigroup S. If $a^n \in$ A, for some odd natural number n then $< ast >^n, < sta >^n < sat >^n \in$ A for all $s, t \in$ T.
THEOREM 3.6 : Every completely semiprime ideal A in a ternary semigroup T is a pseudo symmetric ideal.

Proof **:** Let A be a completely semiprime ideal of the ternary semigroup T.
Let $x, y, z \in$ T and $xyz \in$ A. $xyz \in$ A implies $(yzx)^3 = (yzx)(yzx)(yzx) = yz(xyz)(xyz)x \in$ A.
$(yzx)^3 \in$ A, A is completely semiprime ideal $\Longrightarrow yzx \in$ A.
Similarly $(zxy)^3 = (zxy)(zxy)(zxy) = z(xyz)(xyz)xy \in$ A.
$(zxy)^3 \in$ A, A is completely semiprime ideal $\Rightarrow zxy \in$ A.
If $s, t \in T^1$, then $(xsytz)^3 = (xsytz)(xsytz)(xsytz) = xsyt[zx(syt)(zxs)y]tz \in$ A.
$(xsytz)^3 \in$ A, A is completely semiprime implies $xsytz \in$ A.
Therefore A is a pseudo symmetric ideal.
NOTE 3.7 : The converse of theorem 3.6, is not true, i.e., a pseudo symmetric ideal of a ternary semigroup need not be completely semiprime.
EXAMPLE 3.8 : Let T = $\{a, b, c\}$. Define a ternary operation [ ] on T as $[abc] = a.b.c$ where . is binary operation and the binary operation defined as follows

.	*a*	*b*	*c*
a	*a*	*a*	*a*
b	*a*	*a*	*a*
c	*a*	*b*	*c*

Clearly (T, [ ]) is a ternary semigroup and $\{a\}$, $\{a, b\}$, T are the ideals of T.
Now $aaa \in \{a\} \Longrightarrow aaaaa, ababa, acaca, aaaba, abaca, acaba \in \{a\}$
$abb \in \{a\} \Longrightarrow aabab, abbbb, acbcb, aabbb, abbcb, acbab \in \{a\}$
$baa \in \{a\} \Longrightarrow baaaa, bbbba, bcaca, baaba, bbaca, bcaaa \in \{a\}$
$aba \in \{a\} \Rightarrow aabaa, abbba, acbca, aabba, abbca, acbaa \in \{a\}$
$bab \in \{a\} \Rightarrow baaab, bbabb, bcbcb, baabb, bbacb, bcaab \in \{a\}$
$bbb \in \{a\} \Rightarrow babab, bbbbb, bcbcb, babbb, bbbcb, bcbab \in \{a\}$
$bba \in \{a\} \Rightarrow babaa, bbbba, bcbca, babba, bbbca, bcbaa \in \{a\}$
$acc \in \{a\} \Rightarrow aacac, abcbc, acccc, aacbc, abccc, accac \in \{a\}$
$caa \in \{a\} \Longrightarrow caaaa, cbcba, ccaca, caaba, cbaca, ccaaa \in \{a\}$
$aca \in \{a\} \Rightarrow aacaa, abcba, accca, aacba, abcca, accaa \in \{a\}$
$cac \in \{a\} \Rightarrow caaac, cbabc, ccacc, caabc, cbacc, ccaac \in \{a\}$
$cca \in \{a\} \Rightarrow cacaa, cbcba, cccca, cacba, cbcca, cccaa \in \{a\}$
$abc \in \{a\} \Longrightarrow aabac, abbbc, acbcc, aabbc, abbcc, acbac \in \{a\}$
$bca \in \{a\} \Longrightarrow bacaa, bbcba, bccca, bacba, bbcca, bccaa \in \{a\}$
$cab \in \{a\} \Longrightarrow caaab, cbabb, ccacb, caabb, abacb, ccaab \in \{a\}$.

Therefore $\{a\}$ is a pseudo symmetric ideal in T. Here $b^3 = a \in \{a\}$, but $b \notin \{a\}$.
Therefore $\{a\}$ is not a completely semiprime ideal.

THEOREM 3.9 : If A is a pseudo symmetric ideal of a ternary semigroup T then $A_2 = \{x : x^n \in A$ for some odd natural number $n \in N\}$ is a minimal completely semiprme ideal of T.
Proof **:** Clearly A $\subseteq A_2$ and hence A_2 is a nonempty subset of T. Let $x \in A_2$ and $s, t \in$ T.
Now $x \in A_2 \Longrightarrow x^n \in$ A for some odd natural number *n*. $x^n \in$ A, $s, t \in$ T, A is a pseudo symmetric ideal of T
$\Longrightarrow (xst)^n \in$ A, $(sxt)^n \in$ A, $(stx)^n \in$ A $\Longrightarrow xst, sxt, stx \in A_2$.
Therefore A_2 is an ideal of T. Let $x \in$ T and $x^3 \in A_2$.
Now $x^3 \in A_2 \Longrightarrow (x^3)^n \in$ A for some odd natural number *n*
$\Longrightarrow x^{3n} \in A \Longrightarrow x \in A_2$. So A_2 is a completely semiprime ideal of T.
Let Q be any completely semiprime ideal containing A. Let $x \in A_2$.
Then $x^n \in$ A for some odd natural number *n*. By corollary 3.4, $x^n \in A \Longrightarrow <x>^n \subseteq A \subseteq Q$.
Since Q is completely semiprime, $<x>^n \subseteq Q \Longrightarrow x \in Q$.
Therefore A_2 is the minimal completely semiprime ideal of S containing A.
THEOREM 3.10 : If A is a pseudo symmetric ideal of a ternary semigroup T then
$A_2 = A_4$.
Proof **:** By theorem 2.23, $A_4 \subseteq A_2$. Let $x \in A_2$. Then $x^n \in$ A for some odd natural number *n*.
Since A is pseudo symmetric, $x^n \in A \Longrightarrow <x>^n \subseteq A \Longrightarrow x \in A_4$.
Therefore $A_2 \subseteq A_4$ and hence $A_2 = A_4$.

THEOREM 3.11 : If A is a pseudo symmetric ideal of a ternary semigroup T then $A_4 = \{x : <x>^n \subseteq A$ for some odd natural number $n\}$ is the minimal semiprime ideal of T containing A.

***Proof* :** Clearly $A \subseteq A_4$ and hence A_4 is a nonempty subset of S. Let $x \in A_4$ and $s, t \in T$.
Since $x \in A_4$, $< x>^n \subseteq A$ for some odd natural number n.
Now $< xst>^n \subseteq < x>^n \subseteq A$, $< sxt >^n$ and $< stx>^n \subseteq < x>^n \subseteq A \Rightarrow xst, sxt, stx \in A_4$.
Then A_4 is an ideal of S containing A. Let $x \in T$ such that $< x>^3 \subseteq A_4$.
Then $(< x >^3)^n \subseteq A \Rightarrow < x >^{3n} \subseteq A \Rightarrow x \in A_4$. Therefore A_4 is a semiprime ideal of T containing A.
Suppose P is a semiprime ideal of T containing A.
Let $x \in A_4$. Then $< x >^n \subseteq A \subseteq P$ for some odd natural number n.
Since P is a semiprime ideal of T, $< x>^n \subseteq P$ for some odd natural number $n \Rightarrow x \in P$. Therefore $A_4 \subseteq P$ and hence A_4 is the minimal semiprime ideal of T containing A.

THEOREM 3.12 : Every prime ideal P minimal relative to containing a pseudo symmetric ideal A in a ternary semigroup T is completely prime.

Proof : Let S be a ternary subsemigroup generated by T\P. First we show that $A \cap S = \emptyset$. If $A \cap S \neq \emptyset$, then there exist $x_1, x_2, x_3, \ldots, x_n \in T \backslash P$ such that $x_1 x_2 x_3 \ldots x_n \in A$ where n is an odd natural number. By corollary 3.3, $< x_1 >< x_2 > \ldots\ldots < x_n > \subseteq A \subseteq P$. Since P is a prime ideal, we have $< x_i > \subseteq P$ for some i. It is a contradiction. Thus $A \cap S = \emptyset$. Consider the set $\Sigma = \{ B : B$ is a ideal in T containing A such that $B \cap S = \emptyset \}$. Since $A \in \Sigma$, Σ is nonempty. Now Σ is a poset under set inclusion and satisfies the hypothesis of Zorn's lemma. Thus by Zorn's lemma, Σ contains a maximal element, say M. Let X, Y and Z be three ideals in T such that $XYZ \subseteq M$. If $X \not\subseteq M$, $Y \not\subseteq M$, $Z \not\subseteq M$, then $M \cup X$, $M \cup Y$ and $M \cup Z$ are ideals in T containing M properly and hence by the maximality of M, we have $(M \cup X) \cap S \neq \emptyset$, $(M \cup Y) \cap S \neq \emptyset$ and $(M \cup Z) \cap S \neq \emptyset$. Since $M \cap S = \emptyset$, we have $X \cap S \neq \emptyset$, $Y \cap S \neq \emptyset$ and $Z \cap S \neq \emptyset$. So there exists $x \in X \cap S$, $y \in Y \cap S$ and $z \in Z \cap S$. Now, $xyz \in XYZ \cap T \subseteq M \cap T = \emptyset$. It is a contradiction. Therefore either $X \subseteq M$ or $Y \subseteq M$ or $Z \subseteq M$ and hence M is prime ideal containing A. Now, $A \subseteq M \subseteq T \backslash S \subseteq P$. Since P is a minimal prime ideal relative to containing A, we have $M = T \backslash S = P$ and $S = T \backslash P$. Let $xyz \in P$. Then $xyz \notin S$. Suppose if possible $x \notin P$, $y \notin P$, $z \notin P$. Now $x \notin P$, $y \notin P$, $z \notin P \Rightarrow x, y, z \in T \backslash P \Rightarrow x, y, z \in S \Rightarrow xyz \in S$. It is a contradiction. Therefore either $x \in P$ or $y \in P$ or $z \in P$. Therefore P is a completely prime ideal of T.

THEOREM 3.13 : Let A be an ideal of a ternary semigroup T. Then A is completely prime iff A is prime and pseudo symmetric.

***Proof* :** Suppose A is a completely prime ideal of T. By theorem 2.56, A is prime.
Let $x, y, z \in T$ and $xyz \in A$.
$xyz \in A$, A is completely prime $\Rightarrow x \in A$ or $y \in A$ or $z \in A \Rightarrow xsytz \in A$ for all $s, t \in T$.
Therefore A is a pseudo symmetric ideal.
Conversely Suppose that A is prime and pseudo symmetric.
Let $x, y, z \in T$ and $xyz \in A$. $xyz \in A$, A is a pseudo symmetric ideal
$\Rightarrow <x><y><z> \subseteq A \Rightarrow <x> \subseteq A$ or $<y> \subseteq A$ or $< z > \subseteq A \Rightarrow x \in A$ or $y \in A$ or $z \in A$.
Therefore A is completely prime.

COROLLARY 3.14: Let A be an ideal of a ternary semigroup T. Then A is completely prime iff A is prime and completely semiprime.

***Proof* :** The proof follows from theorem 3.13,

COROLLARY 3.15 : Let A be an ideal of a ternary semigroup T. Then A is completely semiprime iff A is semiprime and pseudo symmetric.

***Proof* :** Suppose that A is completely semiprime. By theorem 2.59, A is semiprime and also by theorem 3.6, A is pseudo symmetric.
Conversely suppose that A is semiprime and pseudo symmetric.
Let $x \in T$ and $x^3 \in A$. $x^3 \in A$, A is pseudo symmetric $\Rightarrow <x^3> \subseteq A \Rightarrow <x> \subseteq A \Rightarrow x \in A$.
Therefore A is a completely semiprime ideal of T.

THEOREM 3.16 : Let A be a pseudo symmetric ideal of a ternary semigroup T. Then the following are equivalent.

1) **A_1 = The intersection of all completely prime ideals of T containing A.**
2) **A_1^1 = The intersection of all minimal completely prime ideals of T containing A.**
3) **A_1^{11} = The minimal completely semiprime ideal of T relative to containing A.**
4) **$A_2 = \{x \in T : x^n \in A$ for some odd natural number $n\}$**
5) **A_3 = The intersection of all prime ideals of T containing A.**
6) **A_3^1 = The intersection of all minimal prime ideals of T containing A.**

7) A_3^{11} **= The minimal semiprime ideal of T relative to containing A.**

8) $A_4 = \{x \in T : \langle x\rangle^n \subseteq A$ **for some odd natural number** $n\}$

Proof : Since completely prime ideals containing A and minimal completely prime ideals containing A and minimal completely semiprime ideal relative to containing A are coincide, it follows that $A_1 = A_1^1 = A_1^{11}$. Since prime ideals containing A and minimal prime ideals containing A and the minimal semiprime ideal relative to containing A are coincide, it follows that $A_3 = A_3^1 = A_3^{11}$. Since A is pseudo symmetric ideal, we have $A_2 = A_4$. Now by corollary 3.15, we have $A_1^{11} = A_3^{11}$. Therefore $A_1 = A_1^1 = A_1^{11} = A_3 = A_3^1 = A_3^{11}$ and $A_2 = A_4$. Hence the given conditions are equivalent.

DEFINITION 3.17 : A ternary semigroup T is said to be a ***pseudo symmetric ternary semigroup*** provided every ideal in T is a pseudo symmetric ideal.

THEOREM 3.18 : Every commutative ternary semigroup is a pseudo symmetric ternary semigroup.

***Proof* :** Suppose T is commutative ternary semigroup.

Then abc = bca = cab = bac = cba = acb for all $a, b, c \in T$. Let A be an ideal of T. Let $a, b, c \in T$, $abc \in A$ and $s, t \in T$. Then $asbtc = abstc = absct = abcst \in A$. Therefore A is a pseudo symmetric ideal and hence T is a pseudo symmetric ternary semigroup.

THEOREM 3.19 : Every pseudo commutative ternary semigroup is a pseudo symmetric ternary semigroup.

***Proof* :** Let T be a pseudo commutative ternary semigroup and A be any ideal of T.

Let $x, y, z \in T$, $xyz \in A$. If $s, t \in T$. Then $xsytz = syxtz = syzxt = s(xyz)t \in A$.

Therefore $xsytz \in A$ for all $s, t \in T$. Therefore A is a pseudo symmetric ideal. Therefore T is a pseudo symmetric semigroup.

THEOREM 3.20 : If T is a ternary semigroup in which every element is a midunit then T is a pseudo symmetric ternary semigroup.

***Proof* :** Let T be a ternary semigroup in which every element is a midunit and A be any ideal of T. Let $x, y, z \in T$ and $xyz \in A$. If $s \in T$, then s is a midunit and hence, $xsysz = xyz \in A$. Hence A is a pseudo symmetric ideal. Therefore T is a pseudo symmetric ternary semigroup.

IV. SEMIPSEUDO SYMMETRIC IDEALS:

DEFINITION 4.1 : An ideal A in a semigroup T is said to be ***semipseudo symmetric*** provided for any odd natural number n, $x \in T$, $x^n \in A \Rightarrow \langle x\rangle^n \subseteq A$.

THEOREM 4.2 : Every pseudo symmetric ideal of a ternary semigroup is a semipseudo symmetric ideal.

***Proof* :** Let A be a pseudo symmetric ideal of a ternary semigroup T.

Let $x \in T$ and $x^n \in A$ for some odd natural number n. Since A is pseudo symmetric, by corollary 3.4, $x^n \in A \Rightarrow \langle x\rangle^n \subseteq A$. Therefore A is a semipseudo symmetric ideal.

NOTE 4.3 : The converse of theorem 4.2, is not true. i.e. a semipseudo symmetric ideal of a ternary semigroup need not be a pseudo symmetric ideal.

EXAMPLE 4.4 : Let S be a free ternary semigroup over the alphabet $\{a, b, c, d, e\}$.

Let $A = \langle abc\rangle \cup \langle bca\rangle \cup \langle cab\rangle$. Since $abc \in A$ and $adbec \notin A$, A is not pseudo symmetric.

Suppose $x^n \in A$ for some odd natural number n. Now the word x contains abc or bca or cab and hence $\langle x\rangle^n \subseteq A$. Therefore $x^n \in A$ for some odd natural number $n \Rightarrow \langle x\rangle^n \subseteq A$. Therefore A is a semipseudo symmetric ideal.

THEOREM 4.5 : Every semiprime ideal P minimal relative to containing a semipseudo symmetric ideal A in a ternary semigroup T is completely semiprime.

***Proof* :** Write $S = \{x^n : x \in T \setminus P$ for any odd natural number $n\}$. First we show that $A \cap S = \emptyset$.

If $A \cap S \neq \emptyset$, then there exists an element $x \in T \setminus P$ such that $x^n \in A$ where n is odd natural number. Since A is a semipseudo symmetric ideal, $\langle x\rangle^n \subseteq A \subseteq P \Rightarrow \langle x\rangle^n \subseteq P \Rightarrow x \in P$. It is a contradiction. Thus $A \cap S = \emptyset$.

Consider the set $\Sigma = \{ B : B$ is an ideal in T containing A such that $B \cap S = \emptyset \}$. Since $A \in \Sigma$, Σ is nonempty. Now Σ is a poset under set inclusion and satisfies the hypothesis of Zorn's lemma. Thus by Zorn's lemma, Σ contains a maximal element, say M. Suppose $\langle a\rangle^3 \subseteq M$ and $a \notin M$. Then $M \cup \langle a\rangle$ is an ideal containing A.

Since M is maximal in Σ, we have $(M \cup \langle a\rangle) \cap S \neq \emptyset$.

Then there exists $x \in T \setminus P$ such that $x^n \in \langle a\rangle \cap S$ for some odd natural number n.

Therefore $x^{3n} \in \langle a\rangle^3 \cap S \subseteq M \cap S \Rightarrow x^{3n} \in M \cap S$. It is a contradiction.

Therefore M is a semiprime ideal containing A.

Now, $A \subseteq M \subseteq T \setminus S \subseteq P$. Since P is a minimal semiprime ideal relative to containing A,

we have $M = T \setminus S = P$. Let $x \in S$, $x^m \in P$. Suppose if possible $x \notin P$.

Now $x \notin P \Rightarrow x \in S \Rightarrow x^m \in S$. It is a contradiction. Therefore $x \in P$.

Hence P is a completely semiprime ideal.

COROLLARY 4.6 : Every prime ideal P in a ternary semigroup T minimal relative to containing a semipseudo symmetric ideal A is completely prime.

***Proof* :** Since every prime ideal is a semiprime ideal, by theorem 4.5, we have P is a completely semiprime ideal and by theorem 2.25, P is a completely prime ideal.

COROLLARY 4.7 : Every prime ideal minimal relative to containing a pseudo symmetric ideal A in a ternary semigroup T is completely prime.

***Proof* :** Let P be a prime ideal containing a pseudo symmetric ideal A of a ternary semigroup T. By theorem 4.2, every pseudo symmetric ideal is a semipseudo symmetric ideal, by corollary 4.6, P is a completely prime ideal of T.

THEOREM 4.8 : If A is an ideal in a ternary semigroup T, then the following are equivalent.

1) A is completely semiprime.
2) A is semiprime and pseudo symmetric.
3) A is semiprime and semipseudo symmetric.

***Proof* :** (1) ⇒(2) : Suppose A is a completely semiprime ideal of T. By theorem 2.59, A is a semiprime ideal of T and by theorem 3.6, A is a pseudo symmetric ideal of T.

(2) ⇒(3) : Suppose A is semiprime and pseudo symmetric. By theorem 4.2, A is a semipseudo symmetric ideal. Hence A is semiprime and semipseudo symmetric.

(3) ⇒ (1) : Suppose A is semiprime and semipseudo symmetric.

Let $x \in T$, $x^3 \in A$. Since A is semipseudo symmetric, $x \in T$, $x^3 \in A \Rightarrow \langle x\rangle^3 \subseteq A$.

Since A is semiprime, by theorem 2.57, $\langle x \rangle^3 \subseteq A \Rightarrow \langle x \rangle \subseteq A$.

∴ A is completely semiprime.

THEOREM 4.9 : If A is an ideal of a semisimple ternary semigroup T, then the following are equivalent.

1) A is completely semiprime.
2) A is pseudo symmetric.
3) A is semipseudo symmetric.

***Proof* :** (1) $\Longrightarrow$(2) : Suppose that A is completely semiprime. By theorem 3.6, A is pseudo symmetric.

(2) $\Longrightarrow$ (3) : Suppose that A is pseudo symmetric. By theorem 4.2, A is semipseudo symmetric.

(3) $\Longrightarrow$ (1) : Suppose that A is semipseudo symmetric. Let $x \in T$, $x^3 \in A$. Since A is semipseudo symmetric, $x^3 \in A \Longrightarrow \langle x \rangle^3 \subseteq A$. Since T is semisimple, x is a semisimple element. Therefore $x \in \langle x \rangle^3 \subseteq A$.

Thus A is completely semiprime.

THEOREM 4.10 : If A is an ideal of a ternary semigroup T, then the following are equivalent.

1) A is completely prime.
2) A is prime and pseudo symmetric.
3) A is prime and semipseudo symmetric.

***Proof* :** (1) ⇒ (2) : Suppose that A is completely prime. By theorem 3.13, A is prime and pseudo symmetric.

(2) ⇒ (3) : Suppose A is prime and pseudo symmetric. Since A is pseudo symmetric by theorem 4.2, A is semipseudo symmetric.

(3) ⇒(1) : Suppose A is prime and semipseudo symmetric. Since A is prime by theorem 2.58, A is semiprime. Since A is semiprime and semipseudo symmetric, by theorem 4.9, A is completely semiprime. Since A is prime and completely semiprime by theorem 2.55, A is completely prime.

THEOREM 4.11 : Let A be a semipseudo symmetric ideal of a ternary semigroup T. Then the following are equivalent.

1) A_1=The intersection of all completely prime ideals of T containing A.
2) A_1^1 = The intersection of all minimal completely prime ideals of T containing A.
3) A_1^{11} = The minimal completely semiprime ideal of T relative to containing A.
4) A_2= {$x \in S$: $x^n \in A$ for some odd natural number *n*}
5) A_3= The intersection of all prime ideals of T containing A.
6) A_3^1 = The intersection of all minimal prime ideals of T containing A.
7) A_3^{11} = The minimal semiprime ideal of T relative to containing A.
8) A_4 = {$x \in S$: $\langle x\rangle^n \subseteq A$ for some odd natural number *n*}

Proof : Since completely prime ideals containing A and minimal completely prime ideals containing A and minimal completely semiprime ideal relative to containing A are coincide, it follows that $A_1 = A_1^1 = A_1^{11}$. Since prime ideals containing A and minimal prime ideals containing A and the minimal semiprime ideal relative to containing A are coincide, it follows that $A_3 = A_3^1 = A_3^{11}$. Since A is semipseudo symmetric ideal, we have

$A_2=A_4$. Now by corollary 4.8, we have $A_1^{11} = A_3^{11}$. Therefore $A_1= A_1^1 = A_1^{11} = A_3 = A_3^1 = A_3^{11}$ and $A_2=A_4$. Hence the given conditions are equivalent.

THEOREM 4.12 : If M is a maximal ideal of a ternary semigroup T with $M_4 \neq S$, then the following are equivalent.
1) M is completely prime.
2) M is completely semiprime.
3) M is pseudo symmetric.
4) M is semipseudo symmetric.

Proof **: (**1) ⇒ (2) : Suppose that M is completely prime. By theorem 2.59, M is completelty semiprime.
(2) ⇒ (3) : Suppose that M is completely semiprime. By theorem 3.6, M is pseudo symmetric.
(3) ⇒ (4) : Suppose that M is pseudo symmetric. By theorem 4.2, M is semipseudo symmetric.
(4) ⇒ (1) : Suppose M is semipseudo symmetric. By the theorem 4.11, $M \subseteq M_4 \subseteq T$. Since M is maximal ideal and $M_4 \neq S$, it implies that $M = M_4$. Let $x \in T$, $x^3 \in M$. Since M is semipseudo symmetric, $\langle x\rangle^3 \subseteq M$. Then $x \in M_4 = M$. ∴ M is completely semiprime.
Let $x, y \in S$, $xy \in M$. Since M is completely semiprime, by corollary 2.24,
$xyz \in M \Rightarrow \langle x\rangle \langle y\rangle \langle z \rangle \subseteq M$. Suppose if possible $x \notin M, y \notin M, z \notin M$. Then $M \cup \langle x\rangle$, $M \cup \langle y\rangle$, $M \cup \langle z \rangle$ are ideals of T and $M \cup \langle x\rangle = M \cup \langle y\rangle = M \cup \langle z \rangle = S$, Since M is maximal, $y, z \in M \cup \langle x\rangle$, $x, z \in M \cup \langle y\rangle$ and $x, y \in M \cup \langle z\rangle \Rightarrow y, z \in \langle x\rangle, x, z \in \langle y\rangle, x, y \in \langle z\rangle \Rightarrow \langle x\rangle = \langle y\rangle = \langle z \rangle$. Now $\langle x\rangle \langle y\rangle \langle z \rangle \subseteq M$
$\Rightarrow \langle x\rangle \langle y\rangle \langle z \rangle = \langle x\rangle^3 \subseteq M \Rightarrow x^3 \in M \Rightarrow x \in M$. It is a contradiction. ∴ either $x \in M$ or $y \in M$.
∴ M is completely prime.

DEFINITION 4.13 : A ternary semigroup T is said to be a ***semipseudo symmetric ternary semigroup*** provided every ideal of T is semipseudo symmetric.

THEOREM 4.14 : A ternary semigroup T is semipseudo symmetric iff every principal ideal is semipseudo symmetric.

Proof **:** Suppose a ternary semigroup T is semipseudo symmetric. Then every ideal of T is semipseudo symmetric. Hence every principal ideal of T is semipseudo symmetric.
Conversely suppose that every principal ideal of T is semipseudo symmetric. Let A be any ideal of T.
For $x \in T$, $x^n \in A$ for some odd natural number n. Since $\langle x^n \rangle$ is a semipseudo symmetric ideal, $\langle x \rangle^n \subseteq \langle x^n \rangle$.
Now $\langle x \rangle^n \subseteq \langle x^n \rangle \subseteq A$ for some odd natural number n. ∴ $\langle x\rangle^n \subseteq A$ for some odd natural number n. ∴ A is a semipseudo symmetric ideal. Hence T is a semipseudo symmetric semigroup.

THEOREM 4.15 : In a semipseudo symmetric ternary semigroup T, an element *a* is semisimple iff *a* is intraregular.

Proof **:** Let T be a semipseudo symmetric ternary semigroup. Suppose an element $a \in T$ is semisimple.
Then $a \in \langle a\rangle^3$. Since T is semipseudo symmetric, $\langle a^3 \rangle$ is a semipseudo symmetric ideal. Thus $a^3 \in \langle a^3 \rangle$
$\Rightarrow \langle a \rangle^3 \subseteq \langle a^3 \rangle \Rightarrow a \in \langle a\rangle^3 \subseteq \langle a^3 \rangle$.
Therefore $a = sa^3t$ for some $s, t \in T^1$ and hence a is intraregular.
Conversely suppose that $a \in T$ is intraregular. Then $a = xa^3y$ for some $x, y \in T \Rightarrow a \in \langle a^3\rangle$.
∴ a is semisimple.

THEOREM 4.16 : If T is a semipseudo symmetric ternary semigroup, then the following are true.
1) $S = \{a \in T : \sqrt{\langle a\rangle} \neq T\}$ is either empty or a completely prime ideal.
2) T\S is either empty or an archimedean subsemigroup of T.

Proof **: (**1) If S is an empty set, then there is nothing to prove. If S is nonempty, then clearly S is an ideal of T.
Let $a, b, c \in T$ and $abc \in S$. Suppose if possible $a \notin S$, $b \notin S$, $c \in S$.
Then $\sqrt{\langle a\rangle} = T$, $\sqrt{\langle b\rangle} = T$ and $\sqrt{\langle c \rangle} = T$. Since $abc \in S$, then $\sqrt{\langle abc\rangle} \neq T$.
Now $T = \sqrt{\langle a\rangle} \cap \sqrt{\langle b\rangle} \cap \sqrt{\langle c\rangle} = \sqrt{\langle abc\rangle} \neq T$. It is a contradiction.
Thus $a \in S$ or $b \in S$ or $c \in S$. ∴ S is a completely prime ideal.
(2) Since S is a completely prime ideal, T\S is either empty or a ternary subsemigroup of T.
Let $a, b, c \in T\backslash S$. Then $\sqrt{\langle a\rangle} = \sqrt{\langle b\rangle} = \sqrt{\langle c\rangle} = T$. Now $b,c \in \sqrt{\langle a\rangle}$, $c, a \in \sqrt{\langle b \rangle}$, $c, a \in \sqrt{\langle c \rangle}$
by the theorem 4.11. $b^n \in \langle a\rangle$ for some odd natural number n. So $b^{n+2} \in TaT \Rightarrow b^{n+2} = sat$ for some $s, t \in T$.
If either s or $t \in S$, then $b^{n+2} \in S$ and hence $b \in S$. It is a contradiction. Hence $s, t \in T\backslash S$.
Now $b^{n+2} = sat \in (T\backslash S)\, a\, (T\backslash S)$. Hence T\S is an archimedean ternary subsemigroup of T.

THEOREM 4.17 : If T is a semipseudo symmetric ternary semigroup, then the following are equivalent.
1) T is a strongly archimedean semigroup.
2) T is an archimedean semigroup.
3) T has no proper completely prime ideals.

4) T has no proper completely semiprime ideals.
5) T has no proper prime ideals.
6) T has no proper semiprime ideals.

***Proof* : (**1) ⇒ (2) : Suppose that T is a strongly archimedean ternary semigroup. By theorem 2.4, T is an archimedean ternary semigroup.
(2) ⇒(3) : Suppose that T is an archimedean ternary semigroup. Let P be any completely prime ideal of T.
Let a∈T, b∈P. Since T is an archimedean semigroup, there exists a odd natural number n such that
$a^n \in TbT \subseteq P \Rightarrow a^n \in P \Rightarrow a \in P$. ∴ T⊆P. Clearly P⊆T. Thus P = T.
∴ T has no proper completely prime ideals.
By theorem 2.56, corollary 2.60, and theorem 4.10; (3), (4), (5) and (6) are equivalent.
(5) ⇒(1) : T has no proper prime ideals. Let a, b ∈ T. Since T has no proper prime ideals, $\sqrt{<b>}$ = T. Now $a \in T = \sqrt{<b>} \Rightarrow a^n \in <b>$ for some odd natural number n. Since T is a semipseudo symmetric semigroup, < b > is a semipseudo symmetric ideal and hence $a^n \in <b> \Rightarrow <a>^n \subseteq <b>$. Thus T is a strongly archimedean ternary semigroup. Hence the given conditions are equivalent.

COROLLARY 4.18 : A commutative ternary semigroup T is archimedean iff T has no proper prime ideals.

***Proof* :** Since T is a commutative ternary semigroup, T is a semipseudo symmetric semigroup. By theorem 4.17, T is archimedean iff T has no proper prime ideals.

THEOREM 4.19 : If M is a nontrivial maximal ideal of a semipseudo symmetric ternary semigroup T then M is prime.

***Proof* :** Suppose if possible M is not prime. Then there exists $a, b, c \in T\backslash M$ such that
$<a> <b> <c> \subseteq M$. Now for any $x \in T\backslash M$, we have $T = M \cup <b> = M \cup <c> = M \cup <x>$. Since $b, c, x \in T\backslash M$, we have $b, c \in <x>$ and $x \in <b>$, $x \in <c>$. So $<b> = <c> = <x>$. Therefore $<b>^3 \subseteq M$, $<c>^3 \subseteq M$. If $a \neq b$, then $a = sbt$ for some $s, t \in T$. So $a \in <s> <b> <t>$. If either $s \in M$ or $t \in M$ then $a \in M$. It is a contradiction.
If $s \notin M$ and $t \notin M$, then $<s> <b><t> \subseteq <b>^3 \subseteq M$. ∴ $a \in <s> <b> <t> \subseteq M$. ∴ a∈M. It is a contradiction.
Thus $a = b$ and hence M is trivial, which is not true. So M is prime.

THEOREM 4.20 : If T is a semipseudo symmetric ternary semigroup and contains a nontrivial maximal ideal then T contains semisimple elements.

***Proof* :** Let M be a nontrivial maximal ideal of T. By theorem 4.19, M is prime.
Let $a \in T\backslash M$. Then $< a > \nsubseteq M$. Since M is maximal, $M \cup < a > = T$.
If $< a >^3 \subseteq M$ then $< a > \subseteq M$ which is not true. So $< a >^3 \nsubseteq M$.
Since M is maximal, $M \cup < a >^3 = T$. Now $M \cup < a > = M \cup < a >^3$.
Therefore $a \in < a >^3$ and hence a is semisimple.

THEOREM 4.21 : Let T be a semipseudo symmetric archimedean ternary semigroup. Then an ideal M is maximal iff it is trivial, and S has no maximal ideals if T = T^3.

***Proof* :** If M is trivial, then clearly M is maximal ideal. Conversely suppose that M is maximal. Suppose if possible M is nontrivial. By theorem 4.19, M is prime. Since T is an archimedean semigroup, by theorem 4.17, S has no prime ideals. It is a contradiction. So M is trivial. If T=T^2, then by theorem 2.51, every maximal ideal is prime and hence T has no maximal ideals.

THEOREM 4.22 : Let T be a semipseudo symmetric ternary semigroup containing maximal ideals. If either S has no semisimple elements or T is an archimedean semigroup, then T ≠ T^3 and T^3=M^* where M^* denotes the intersection of all maximal ideals.

***Proof* :** Suppose that T has no semisimple elements. Then by corollary 4.20, every maximal ideal is trivial. So if M is maximal, then $T = M \cup \{a\}$, $a \notin M$. Suppose $a \in T^3$.
Then $a \in T^3 \Rightarrow a = bcd$ for some $b, c, d \in T$.
If $b \neq a$ then $b \in M$ and hence $bcd \in M$ (Since M is Maximal) ⇒ $a \in M$. It is a contradiction. ∴ $b = a$. Similarly we can prove $c = a$ and $d = a$. ∴ $a = bcd = a^3$. ∴ a is semisimple.
It is a contradiction. ∴ $a \notin T^3$. ∴ $T \neq T^3$ and $T^3 \subseteq M$. Let $t \in M^*$ and $t \notin T^3$.
Let $a \in T\backslash\{t\} \Rightarrow ars \neq t, ras \neq t, rsa \neq t$ for all $r, s \in T \Rightarrow ars, ras, rsa \in T\backslash\{t\} \Rightarrow T\backslash\{t\}$ is an ideal. Then T\{t} is a maximal ideal. Hence $t \in T\backslash\{t\}$, it is a contradiction. ∴ $M^* \subseteq T^3$. ∴ $T^3 = M^*$.
Now suppose T is an archimedean ternary semigroup. Since T has maximal ideals, by theorem 4.21, T ≠ T^3. Suppose if possible $x \in T^3\backslash M^*$. Then there exists a maximal ideal M, such that $x \notin M$. So by theorem 4.21, M=T\{x}. Since $x \in T^3$, $x = rst$ for some $r, s, t \in T$. If either r or s or t ∈M, then $x \in M$. It is a contradiction. Therefore $r = s = t = x$ and hence $x = x^3$. Let $a, b, c \in T$, $abc \in M$. Suppose if possible $a \notin M$, $b \notin M$, $c \notin M$. Then $a = x$, $b = x$, $c = x$. Therefore $abc = xxx = x \notin M$. It is a contradiction. Thus M is prime. By theorem 4.17, S has no proper prime ideals. It is a contradiction. Thus $T^3 \subseteq M^*$. As above, we can show that $M^* \subseteq T^3$. Therefore $T^3 = M^*$.

COROLLARY 4.23 : Let T be a commutative ternary semigroup containing maximal ideals. If either T has no idempotents or T is an archimedean ternary semigroup, then $T \neq T^3$ and $T^3=M^*$.

Proof **:** Suppose that T has no idempotents. If T contains a semisimple element *a* then *a* is regular. Hence there exists and element $x, y \in T$ such that $axaya = a$. Now *axaya* is an idempotent in T. It is a contradiction. So T has no semisimple elements. Then by theorem 4.22, we have $T \neq T^3$ and $T^3 = M^*$.

V. Conclusion

Anjaneyulu. A initiated the study of pseudo symmetric ideals in semigroups, Madhusudhana Rao. D, Anjaneyulu. A. and Gangadhara Rao. A. initiated the the study of theory of Γ-ideals in Γ-semigroups and V. B. Subrahmanyeswara Rao Seetamraju, Anjaneyulu and Madhusudhana Rao initiated the study of theory of ideals in partially ordered Γ-semigroups and hence the study of ideals in semigroups, Γ-semigroups and partially ordered Γ-semigorups creates a platform for the pseudo symmetric ideals in ternary semigroups.

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